The Remarkable Diagrams of Johann Maass

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Research into the history of logical diagrams is still insufficient. Thus, quite a number of diagram systems from the past are unknown today. One of these systems was invented in 1793 by Johann Maass. Since his diagrams have several interesting features they are worthy of rediscovery.

1 Introduction

In 1793 Johann Maass published his *Grundriss der Logik* (Outline of Logic). In this book which was designed for logic courses and was quite popular at that time Maass constructed a number of diagrams which he claimed to be an improvement on the well-known Euler diagrams. These diagrams which have several interesting features have quite unjustly been forgotten in the history of logical diagrams .

Johann Gebhard Ehrenreich Maass was born in 1766 as the son of an erudite preacher in Krottendorf, a small village near Halberstadt about 150 kilometers to the southwest of Berlin. In 1784 he began his university studies in theology and philosophy at Halle where he became *Privatdozent* (University Lecturer) in 1791 and a full professor in 1798. Between 1805 and 1806 Maass became deputy vice-chancellor of the university of Halle for the first time. He held this office in two further terms: between 1816 and 1817 and from 1821 onwards. Maass had a weak constitution his whole life. In 1823, aged 57, he died in Halle.

At the beginning of his academic career Maass was influenced by the Leibniz-Wolffian school but later he tended more and more towards Kantianism. He lectured on philosophy, mathematics, and rhetoric. In each subject Maass wrote an outline: in 1796 *Grundriss der reinen Mathematik*, in 1798 *Grundriss der Rhetorik*, and—as early as 1793—the *Grundriss der Logik*. But Maass' fields of interest were much larger. In 1808 he wrote an outline of natural rights and a few treatises in psychology. Furthermore he published a collection of novels in four volumes in 1814.

The most successful work, however, was Maass' *Grundriss der Logik* of which a fifth edition appeared already in 1836. This success was due to the clear formal construction of the work consisting of 410 paragraphs on only 341 pages and to Maass' attempt at enlarging the traditional logic by a *logic of questions*. Of course, Maass was very proud of his diagrams, as can be seen from his preface, where he wrote:

> Above all I am awaiting the judgement of the experts on the new way of illustrating the relationships among terms, categorical propositions, and syllogisms through drawings.[1]

He criticized two earlier authors, who had also constructed diagrams to represent syllogisms, namely Leonhard Euler (1707–1783) and Johann Lambert (1728–1777). The critique of Euler was one sentence long: "The Eulerian invention is not useful"[2]. The critique of Lambert's diagrams, however, was a little more detailed. Maass criticized Lambert's use of two incompatible metaphors. The first metaphor was that the extension of a term corresponds to the extension of a line in his diagrams. The other was that there are two different lines in the diagram to represent two terms in a proposition like *All A are B*, whereas the extensions of *A* and *B* are—at least in the part of *A*—identical.

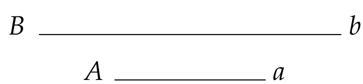
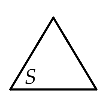


Figure 1
All *A* are *B*.

2 Maass' diagrams

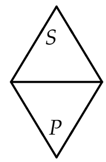
Maass' diagrams can be found on the picture plate at the end of his book. There are in sum eighteen diagrams. Even though he criticized the Euler diagrams (subsequently "EDs"), Maass' own diagrams (subsequently "MDs") are based on the same two principles as Euler's:

1. Every area within a closed curve represents the extension of a term.
   Thus the following MD represents the extension of the term *S*:



2. The relationships between terms are asserted by means of the same relationships between the areas which represent them.
   The following diagram represents, e. g., two terms, *S* and *P*, which have no element in common. The position of the letters marking the terms in a MD is always close to the vertices of the triangles:

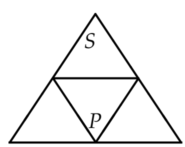
[1] [Maass, 1793], IX. (All quotations are translated by the author.)
[2] Ibid.



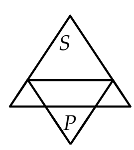
There are, however, two differences between MDs and EDs. The first is that Maass uses triangles to represent extensions of terms, whereas Euler uses circles. The second is that for every categorical proposition there is exactly one MD but a various number of EDs.[3]

The dotted lines in the MDs indicate possible states of affairs. In other words, the dotted lines are *possible lines*. They represent the different relationships between *S* and *P* which can be represented by a diagram. Therefore the MDs for the *a*-, *i*-, and *o*-proposition suitably unify several EDs. Consider the proposition *SoP*. In this state of affairs the three relationships between *S* and *P* are represented by three different EDs. The MD for *SoP*, however, includes all these three relationships:

1. *P* is totally included in *S* ($P \subset S$), i. e., the bottom line has to be solid:



2. *S* and *P* have at least one representative in common ($S \cap P \neq \emptyset$) whence the dotted line in the middle of the MD has to be solid:



[3] It is often said that Euler used only one diagram per proposition, and that the French logician Joseph Gergonne (1771–1859) was the first who showed that there has to be more than one diagram for the propositions *a*, *i*, and *o*. This common view is wrong on both counts. First, Euler used his diagrams like Gergonne except for the diagram for identity which Euler didn't have (this defect will be ignored below). Secondly, it was Karl Krause (1781–1832) who completed Euler's diagram system. Cf. [Krause, 1803].

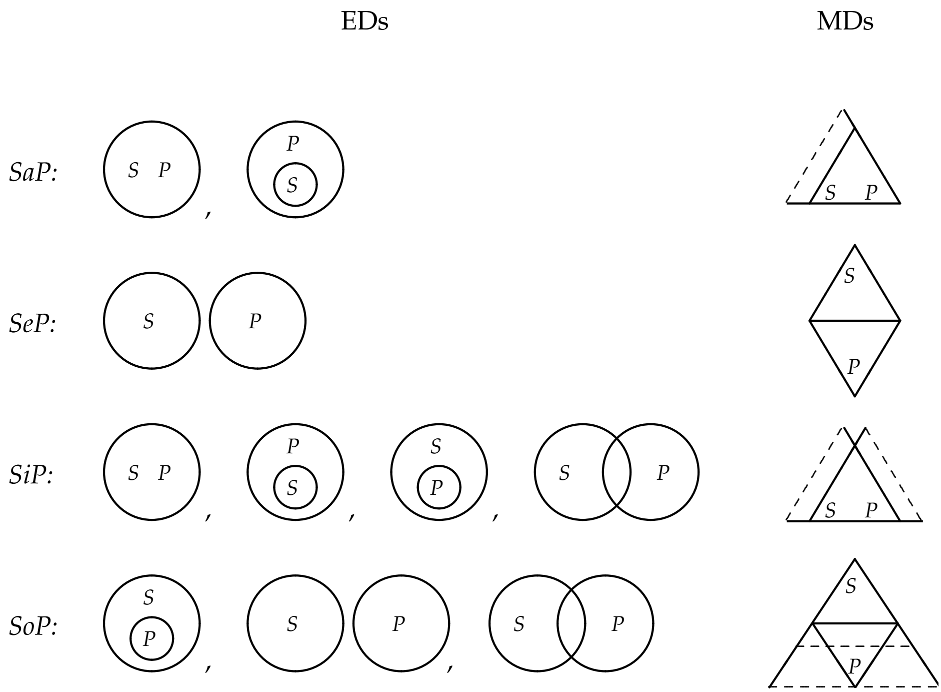
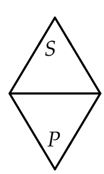


Figure 2
EDs and MDs for the four categorical propositions

3. *S* and *P* have no representative in common ($S \cap P = \emptyset$) which means that there is no dotted line:



Using dotted lines in this way, however, is not an invention by Maass. It was Johann Lambert in his *Neues Organon*[4] who first used this method for his line diagrams.[5] We can therefore say that the MDs are a fusion of the principles of

[4] Cf. [Lambert, 1764], 111*ff.*

[5] It is a widely held opinion that William Thomson (1819–1890) first used area diagrams with dotted lines to represent uncertainties in the categorical propositions. But his contribution in the second edition of his *Laws of Thought* (1849)—about fifty years after Maass' outline—is very modest. In fact

Euler (representing extensions of terms as areas) and Lambert (using dotted lines to represent possible states of affairs).

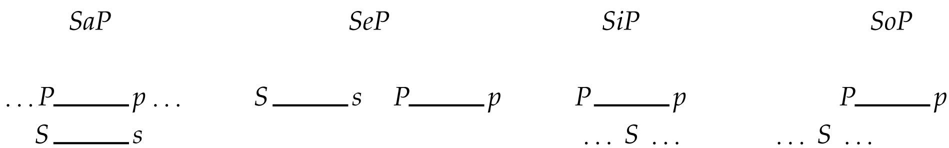
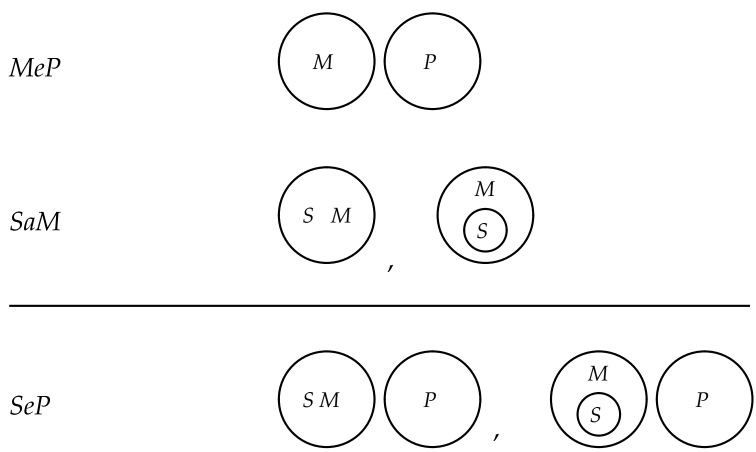
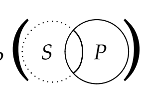
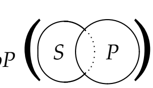


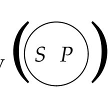
Figure 3
Lambert's line diagrams

3 Solving syllogistic problems

To examine the validity of a syllogism with the help of EDs one has to draw the diagrams for the two premises first and then to combine them. The conclusion is valid if it holds for all the possible combinations. Consider *celarent*:

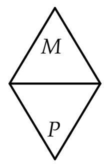


it is limited to two diagrams, the first for *SiP*  and the second for *SoP* .

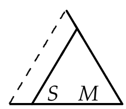
The defect of the Eulerian system—the lack of a diagram for identity —was not removed by Thomson.

The drawback of the Eulerian system is the fact that there is no simple algorithm to get the total number of all the possible conclusion diagrams. Using MDs, however, one just has to bring together the premise diagrams to get exactly one diagram which represents the conclusion. Consider *celarent* again:

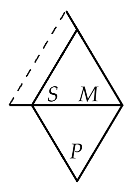
MeP



SaM

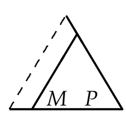


SeP

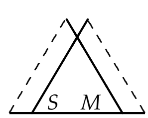


However, the solution for *celarent* is an exception. Consider, for example, the premises for *darii*:

MaP

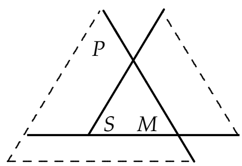


SiM

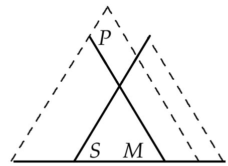


?

Which diagram follows? Obviously several combinations are possible which are not equivalent to one another, e. g.,



or



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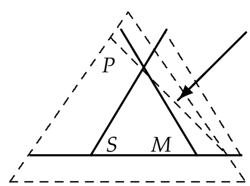
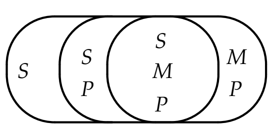
Here the weakness of MDs becomes clear: One has to know the solution of the syllogism *before* the correct diagram for the given premises can be constructed. Otherwise it would be very hard to find the correct diagram of *darii*:



The dotted line marked by the arrow is necessary for the case in which all the *Ps* which are not *M* are *S*. Such difficulties in construction are not a problem of MDs only. Admittedly, the conclusion diagram looks very sophisticated but one has to consider that the solution using EDs consists of a list of 16 (!) different diagrams.[6]

4 Geometrical features

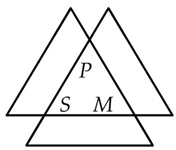
In the case which is in the conclusion diagram for *darii* marked with the arrow the unusual triangle form of MDs turns out to be superior to the circle form of EDs. This is because it is impossible to use circles for constructing this state of affairs:[7]



[6] Johnson-Laird found in a psychological test that none of his test persons found all of the sixteen possible EDs for *darii*. See [Johnson-Laird, 1983], 117*f.*

[7] The triangle form was not an invention of Maass. The oldest diagram using areas at all was a triangle diagram in Juan Vives' *De censura veri* (1555).

This example illustrates an interesting feature of logical diagrams as such. The expressive range of a diagram system depends on the geometrical form the diagrams have. E. g., the triangle form of MDs has more expressive power than the circle form of EDs. Therefore it is very easy to find more MDs which cannot be reconstructed with the help of circles[8], e. g.,



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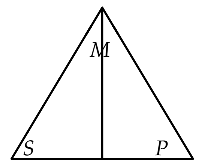

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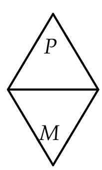
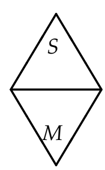
5 MDs and visual information

Maass, however, does not deal with such problems because he does not use these diagrams to solve syllogistic problems. They serve him to illustrate the two "general laws" of logic, "from two particular premises nothing follows", and "from two negative premises nothing follows".[9] This allows Maass to avoid the diagrammatic problems. Consider the law that no conclusion follows from two negative premises. In the case of two *e*-premises only two premise diagrams are possible:



and



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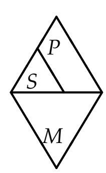
These diagrams hold for two different but incompatible conclusion diagrams.

The first in which all *S* are *P*,



[8] Without removing the principle of representation. Obviously the discussed states of affairs can be represented by the well-known Venn diagrams.

[9] Maass, loc. cit., 304*f.*

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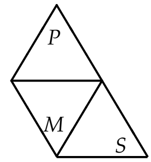
and the second in which no *S* is *P*:  .

Maass doesn't examine the complicated *SoP*-diagrams, however. Instead he refers to the law of subalternation that everything which holds for *SeP* holds for *SoP* a fortiori. Therefore nothing can be deduced from two negative premises.

Both laws described above follow from the *rules of distribution*. A term is said to be distributed if it is actually applied to all the objects to which it can refer, that is, if it is used in its full reference or extension. A term is undistributed if it is explicitly applied only to a part of the objects to which it could refer. The following table holds for the four categorical propositions.

Table 1
Distribution of terms

	S	*P*
SaP	distributed	undistributed
SeP	distributed	distributed
SiP	undistributed	undistributed
SoP	undistributed	distributed

The distribution of the terms can be directly taken from the MDs. Every term which is represented by a triangle with solid lines and only such lines is distributed, and triangles with dotted lines represent undistributed terms. This is due to the strict correspondence between triangles and the information about the terms we have. If we are sure about the position of every representative of a term—i. e. if the term is distributed—the triangle cannot have any dotted line. In other words, dotted lines indicate our uncertainty about the exact border of a triangle, i. e. about the extension of a term.

Representing the information content of a categorical proposition, however, is a modern principle of logic: "[The] informational perspective is part of the modern, semantic approach to logic associated with names like Gödel, Tarski, Robinson, and the like. On this view, a purported rule of inference is valid or not depending on whether it in fact guarantees that the information by the conclusion is implicit in the information represented by the premises."[10] From this point of view the principle of distribution can be understood, and—with the help of MDs—illustrated. A proposition which distributes a term contains obviously more information about this term than a proposition which does not distribute

[10] [Barwise/Etchemendy, 1995], 180.

this term. Therefore the rule of distribution that a term cannot be distributed in the conclusion if it is undistributed in the premises has to be valid since the conclusion must not contain more information than the premises. Explaining this reason with MDs means that a triangle with solid lines in the conclusion cannot have dotted lines in the premises, and this is immediately clear.

6 Conclusion

It has been shown that the diagrams of Johann Maass provide an example of the relationship between the geometrical form and the expressive power of logical diagrams. In this way these diagrams extended our comprehension for syntactical features of diagram systems at all.

Today, the debate on logical diagrams focuses on the concepts of modern logic like *completeness*, *soundness*, *deductiveness*, etc. The diagrams of Johann Maass show that this focus is too restrictive because they use the traditional principle of distribution to represent the categorical propositions from an informational point of view. In this respect, however, they are of topical interest since the informational perspective is one of the basics of modern logic.

Acknowledgements

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